Research on Optimal Power Based on Physical and Differential Equation Models

Jack Mo*, Hao He, Eric Zhang

Sichuan University Pittsburgh Institute, Sichuan, Chengdu, 610227

*Corresponding author: mohaozhou0304@163.com

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Abstract: To make the most scientific arrangement of the physical strength of the cyclists in the race, our team has developed a model that determines the relationship between the rider's position on the course and the power the rider applies. In the paper, we describe in detail the process of our model from the initial idea to the final use, including three times of modification and optimization. At first, we started from the ideal environment and obtained the prototype of the model by simulating the rider's constant movement on the flat and straight track with no wind resistance, and added the influence of the rider's sprint based on uniform speed to obtain the initial power curve. We built a function model of time T on power and change distance using many physical and dynamic principles by constructing the function relationship between power and time and change distance. Then, we obtained the optimal solution through a genetic optimization algorithm. Next, we develop progressively optimized models based on the rudimentary model to make our model more closely match the power output of the actual race. In Model optimization 1, We add the influence of athletes' physical exertion on riding by introducing a punish coefficient which logistic model and standardization ideas guide, based on data. This coefficient will have a constraint on the athletes' real-time power. In Model optimization 2, We pay attention to the velocity component of wind along the direction of the athlete's speed. We add the influence of it by force analysis. This model clearly shows that when the velocity component is the same as the direction of the athletes' speed, the minimum time will be smaller, and the ideal power will be smaller. If not, it will have the opposite effect. In Model optimization 3, we add the influence of bicycle steering on velocity by a limit for maximum speed. There is a relationship between the velocity of the athlete and the tilt angle. So, we explore it by differential equation modeling. In a word, our model considers the differences in areas of expertise and physical fitness among cyclists of different types and genders. We define the power profile at the model preparation stage. After the sensitivity analysis test, our model has strong robustness and accurate simulation results

1. Introduction

More and more sports enthusiasts have favored the road bicycle race since its emergence [1]. Such a race compares physical factors such as the rider's endurance and explosive power and closely relates to the team's operation strategy and physical strength allocation [2]. Therefore, it is used to help cyclists achieve better results beyond their own and record without changing their endurance and explosive power. They need to rely on more scientific and accurate sports strategy decisions.

To solve this problem, our team will give the relationship between the rider's position on the track and the power exerted by the rider to achieve the most scientific exercise planning and help the rider achieve good results.

2. Data preparation and analysis

The Time Trial Specialist is well-round and usually acts as the leader of the team [3]. They tend to have better stamina throughout the race. The climber is different from it. To resist gravity, it is essential to be lightweight in the uphill race, such as the body of the climber, the weight of equipment, and the shape of the body. After searching data about the rider participants, we defined the power profile of time trial specialists and climbers of different genders. The process uses the Curve Fitting Tool with exponential fitting in Matlab to connect the data to the power curve.

3. Method

3.1. The initial model

Cyclists' power is affected by many factors such as physical conditions, the slope, direction, the direction of the wind, elevation, road traffic, and other factors [4, 5]. In order to maximize the simplification of the model, our team initially assumed that the bicycle was running on a flat, straight track with no wind resistance and assumed that the total physical strength of the athlete was constant. Moreover, to achieve the expectation of the fastest and shortest time, it is assumed that the athlete will run out of energy at the finish line. In other words, the work done by the cyclist should be the same.

Since the wind resistance is ignored and the road is flat, the resistance F suffered by the cyclist is constant. According to the following calculation formula:

$$P = F * V \tag{1}$$

$$W = F * S \tag{2}$$

It can be known that when the athlete's physical strength W is constant, the distance she travels is also constant, which is consistent with the actual situation.

When we assume that the cyclist's speed is constant, we can get the simplest case, in which the cyclist's power is a constant relative to time, and the integral is a constant value, namely the physical strength of the athlete. It gets a little bit more complicated when we assume that the athlete is moving at a constant speed, but sometimes he makes an accelerated sprint for a while and then comes back to a constant speed. So, we get a curve of the athlete's power with a bump over time with a bit of bump on the flat line.

If we assume that the initial burst is instantaneous, it can be concluded that the athlete starts with the maximum power and then slowly decays to the normal level with the least time.

3.2. The rudimentary model

Inspired by the initial model, we decided to use differential equation modeling to figure out the speed provided by the cyclist at any moment according to the power of the cyclist at that moment and the mechanical relations among the parameters of the track slope, the force exerted by the cyclist, speed and acceleration. Then, the rider's power at each position can be obtained by introducing time parameter T to match the force with speed, acceleration, and track position.

The rolling friction force plus slope pulling force suffered by the bicycle during the movement can be expressed by the following formula:

$$Frg = g \times (mbick + mrider) \times (cr \times \cos\beta + \sin\beta) \tag{3}$$

To simplify the results, let's ignore the wind speed here:

$$Vwind = 0 \tag{4}$$

By the power formula $P = f_{total} \times V$, we find the power as a function of speed:

$$P = Cm \times V \times (Cd \times A \times \rho/2 \times V2 + Frg + V \times CrVn) \tag{5}$$

In order to solve this power equation for velocity V, we write it in the implicit form:

$$V^3 + 2V^2 \times (0 + \frac{C_r V_n}{C_d \times A \times \rho} + V \times (0 + \frac{2F_{rg}}{C_d \times A \times \rho}) - \frac{2P}{C_m \times C_d \times A \times \rho} = 0 \tag{6}$$

So, we can use the cardanic formulae to obtain the solutions:
If $a^2 + b^3 \geq 0$:

$$V=\sqrt[3]{a+\sqrt{a^2+b^3}}+\sqrt[3]{a-\sqrt{a^2+b^3}}+(\frac{2}{3}\times\frac{C_rV_n}{C_d\times A\times\rho}) \tag{7}$$

If $a^2+b^3\leq 0$ (casus irreducibles, in case of sufficient downhill slope or tailwind speed):

$$V=2\times\sqrt{-b}+\cos(\frac{1}{3}\times\arccos(\frac{a}{\sqrt{-b^3}}))-\frac{2}{3}\times(\frac{C_rV_n}{C_d\times A\times\rho}) \tag{8}$$

$$a=\frac{C_r^3V_n}{27}+\frac{2F_{rg}C_rV_n}{3(C_d\times A\times\rho)^2}+\frac{P}{C_m\times C_d\times A\times\rho} \tag{9}$$

$$b=\frac{2}{9\times C_d\times A\times\rho}\times(3\times F_{rg}-\frac{2C_rV_n}{C_d\times A\times\rho}) \tag{10}$$

Using the Genetic Optimization Algorithm method, we get the functional image of $t=f(P)$ for every $\Delta S = 1000m$. We need to get the cyclist going at 1200 watts to get the least amount of time.

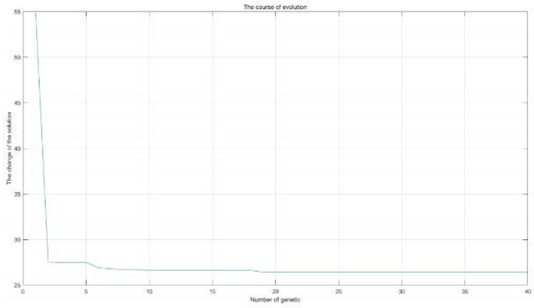

Figure 1 Genetic algorithm process.

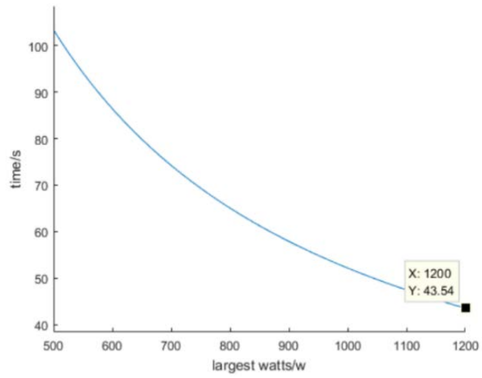

Figure 2 The relationship between Watts and t when $\Delta S = 1000m$.

4. Results and discussion

The rudimentary model is built based on the idea of the original model using physical and mechanical relations and combined with relevant athlete competition data. Compared with the former, it is closer to the actual situation and has specific theoretical support, which is scientific.

However, the current model still has great limitations. We idealize too many conditions, leading to a particular gap between the model and the reality. Next, we need to gradually introduce more restrictions on the secondary basis to optimize the model to adapt to more situations.

4.1. Add the influence of athletes' physical exertion on sports

Prolonged anaerobic breathing causes lactic acid to build up in the body. When lactic acid reaches a certain level, it inhibits movement called the lactate threshold. The Lactate Threshold (LT), also known as the anaerobic Threshold, is the critical intensity at which cycling becomes difficult, breathing becomes heavy, legs start to burn, and the RPE is around 17. Cycling at lactate threshold intensity will only last an hour or so at most for even the best athletes, and the more pedal past it, the less time it can last.

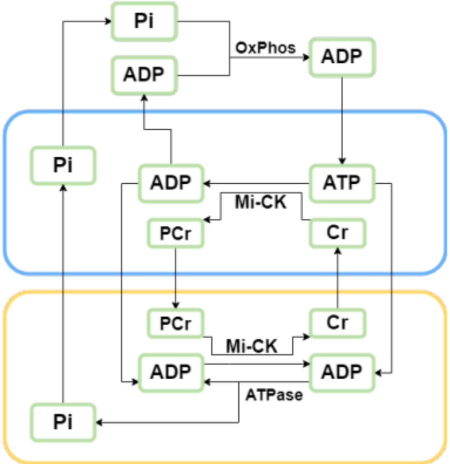

Figure 3 Human movement energy generation.

Functional Threshold Power (FTP) in riding training refers to the maximum average power obtained during stable riding with total effort within one hour. Theoretically, it can maintain about 1 hour under FTP, and as long as the power exceeds FTP for some time, lactic acid in the body will accumulate rapidly, and the maintenance time will be shortened. On the contrary, it can be sustained for a longer time when riding below FTP. The farther away from FTP, the longer it lasts.

We get the idea that high levels of exercise power and long periods of exercise can have negative feedback on cyclists' exercise. The Logistic growth curve is shown below:

$$P(t)=\frac{K\times P_0\times e^{rt}}{K+P_0(e^{rt}-1)} \tag{11}$$

Where P0 is the initial value, K is the final value, and R measures how fast the curve changes. Since the greater the motion power is, the stronger the motion inhibition is, we assume that r and pare proportional to each other, and in order to make the inhibition coefficient between 0.5 and 1, we construct the inhibition coefficient as follows:

$$\alpha=\frac{P_{\max}-\dfrac{P_{\max}\times P_{ftp}\times e^{p^2}}{P_{\max}+P_{ftp}(e^{p^2}-1)}}{P_{\max}-P_{ftp}} \tag{12}$$

Where Pmax is the maximum power of a cyclist's sprint, and Pf tp is the power the rider reaches the lactate threshold. We can find them from big data. P is multiplied by the inhibition coefficient α and re-calculated by the genetic optimization algorithm, and the results are as follows:

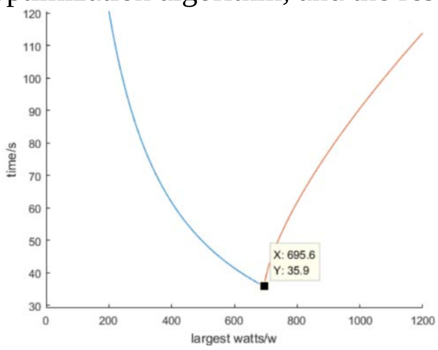

Figure 4 The relationship between Watts and t when ΔS = 1000m.

The optimized model is not difficult to find that athletes can harvest the shortest time when maintaining an average maximum power of 695.6W per kilometer.

4.2. Add the influence of wind speed on velocity

To reduce the complexity of the function, the influence of wind speed is ignored and assumed that Vwind = 0 before. Now, to increase the accuracy of the model and complete its further modification, we consider the influence of wind speed based on the previous model and assume that Vwind is a constant.

Firstly, we need to be clear that the wind direction is uncertain and can be at any angle to the direction of motion. We conducted an orthogonal decomposition of wind speed in the direction of bicycle speed and the direction perpendicular to bicycle speed.

So, we get that the effective wind speed impeding the cycling movement is $V_a = V_{wind} \times cos\gamma$, where γ is the Angle between the wind speed and the athlete. We substitute V_a for V_{wind} and substitute the previous formula to get the new relationship:

$$V^3+2V^2\times(0+\frac{C_r V_n}{C_d\times A\times\rho}+V\times(V_a^2+\frac{2F_{rg}}{C_d\times A\times\rho})-\frac{2P}{C_m\times C_d\times A\times\rho}=0 \qquad (13)$$

Using the genetic optimization algorithm, we made the images of athletes' average power and time per 1000m under different wind speeds.

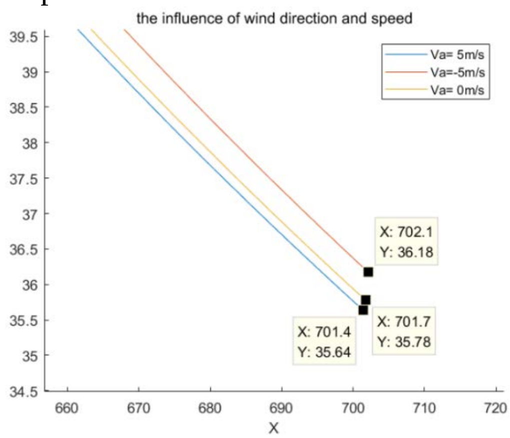

Figure 5 The influence of wind direction and speed.

4.3. Add the effect of bicycle steering on velocity

Our model is more and more appropriate to the actual situation after adding the influence factors of athletes' physical strength and wind speed. However, the cycling road race cannot ignore another important factor: turning.ăăNext. We will focus on turning and further refining the model.

According to the data, we know that most road bicycle competitors use tilting the body center of gravity to turn. Although this method greatly reduces the speed loss when turning, it has specific requirements on the rider's skills, riding speed, and road width. Starting with this limitation, we first assumed that the rider in our study turned with an incline, with no additional energy loss.

According to the data, the ultimate tilt Angle of the rider is 28.07 degrees, and the image obtained by the model shows that for safety, the maximum speed of the rider through the bend should not exceed 7.2m/s.

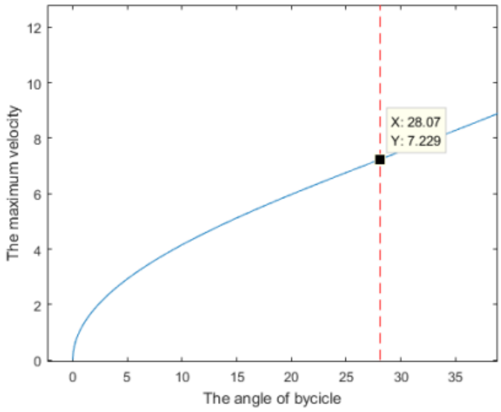

Figure 6 The curve of tilt angle and maximum speed.

4.4. Sensitivity analysis

(1). Sensitivity analysis of rider deviations from the target power distribution

It is unlikely that a rider can follow a highly detailed plan and miss the power targets. So, we cannot get the best power output for every distance interval of the athlete. In this case, a sensitivity analysis towards the error of the output power of the rides is necessary. Based on the distance parameter, we increase and decrease the output power by 1% and 2%, respectively, to obtain five different types of power profiles. Based on this, we apply the most optimal model again and combine the results of the initial in the rudimentary model to calculate the error table of them as follows:

Table 1. Short cut keys for the template

Error/%	Time/s	Deviation times/s	Rate/%
1	36.5	0.8	2
2	37.1	1.3	3.6
-1	36.4	0.6	1.7
-2	37	1.2	3.4

From the above table, we can observe that when the output power increases and decreases 1% and 2%, respectively, the final error of all shapes is within 5%. Therefore, it shows that our model has good robustness. It gives us some enlightening instructions about how to improve our model next.

(2). Sensitivity analysis of the parameter distance in the rudimentary mode

In our model to determine the shortest race time, we treat the distance as a parameter for a functioning relationship between race time and riders' power. The minimum race time corresponding to the model will change if the parameter is different. In order to know how parameter distance affects the model, the sensitivity is carried out. Here we focus on determining the influence of this parameter. The results are as follows:

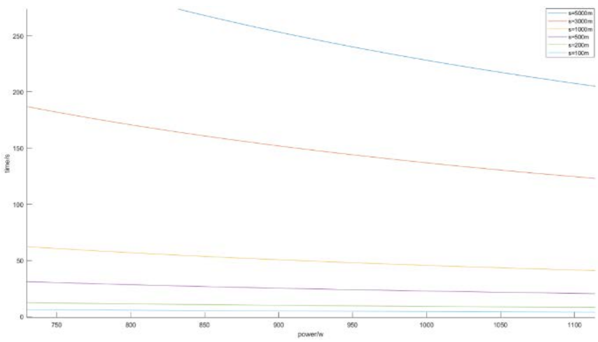

Figure 7 Different parameters correspond to model status.

As shown in this figure, we select several discrete distance parameters. The ideal power has no obvious relationship with power, and the minimum race time varies with the change of the distance parameter. It is consistent with our rudimentary model. Overall, the model is sensitive to minimum race time, which is also confirmed with life experience.

(3). Sensitivity analysis of the external parameter (wind direction and wind speed)

When the wind speed varies from -5m/s to 5m/s, the model has excellent simulation results that can show the ideal power and the minimum race time. It indicates that our model is highly adaptable. At the same time, it is not appropriate to consider excessive wind speed. The hold of Cycling events depends on the weather. So, we are confident that this model is stable and accurate regarding the wind direction and wind speed.

The other external parameters, such as air density, the coefficient of friction, acceleration of gravity, etc., work the same as the wind speed. The equations calculated by differential equation modeling are accurate. They are correlated with our target function and work as independent variables. Above all, our model has strong robustness and accurate simulation results.

5. Conclusion

Our team has analyzed the rider's physical forces for power limits and terrain changes of the track. According to the power of the rider at some point, and the circuit parameters such as slope, the rider weight, velocity, and acceleration between the mechanical relationship, and the rider at the moment provides traction/acceleration, by introducing time parameter t make force and match the position of the track, speed/acceleration obtained rider power should be taken in each position.

In our initial model, we find that Aerodynamics plays a crucial role in timing tests. Air resistance is proportional to the third power of speed. When you reach a moderate speed of 29 kilometers per hour, it accounts for 85 percent of the total force you receive. Moreover, the faster you go, the larger power you have to spend. At the same time, the rider's own body takes on most of the drag, and adopting a more flexible position allows you to ride faster with little effort. To minimize your frontal windward area, flatten your back, and tuck your arms and elbows in. Make sure you do not set it too low to maintain your posture. More importantly, following the front rider is vital to save energy in the road race. The distance between road riding and the car is usually about 15 to 30 cm. If the wind speed is high, the distance to follow the vehicle in front should be small. When a rider is riding behind someone else, it can reduce his energy expenditure by using the eddy currents generated when the rider in front of him breaks through the air resistance to propel the bike forward.

In our model of the relationship between the power of riders and race distance, always keeping a high speed to drive is not a wise choice. The reason is that our body cannot keep a high power above FTP. The lactate threshold is a physiological limit of lactic acid. It refers to the body's maximum capacity to produce acid. It is an important indicator of movement. In cycling theory, FTP during cycling is power at our lactate threshold. Lactic acid builds up, causing our heart rate to increase and our breathing to become faster, preventing us from continuing to keep so that we have to spend time slow speed for relaxing. In order to get a good place in the race, it is important to know your FTP first so that you can assess your riding strength and start your training at different power levels. A sense of urgency drives you to speed up, but it can be difficult to maintain your pace in a large group of people. If you do not stay calm and blindly speed up, you will not realize your rhythm is off, or your heart rate is too high until it is too late. When a race is too long, a good warm-up is a fundamental premise - keep the speed at a low intensity for the first few kilometers, try to increase the intensity gradually after you have established your own pace.

Sharp turns are essential and decisive during a time trial course to this competition. It plays an essential role in all kinds of racing games. The speed should be controlled before turning, and the tilt angle of the speed should be appropriately controlled according to the speed and the size of the turn, while the maximum tilt Angle should not exceed 28.07 degrees. On rainy days, slippery sections should pay more attention to the tilt angle due to friction factors. Otherwise, there will be a risk that the vehicle will skid and fall. In addition, we should pay attention to the riding conditions of other riders to avoid them overtaking us in corners and collisions.

Uphill and downhill should also be taken into consideration. Stop slope riding to maintain the regular pedal action should not suddenly force. We suddenly accelerate only when trying to get rid of an opponent and when tactical needs arise. At the same time, we should also take into account the recovery of the physical strength of the athletes. It can also stand up from the seat so that our gravity can provide part of the force needed. The body should lean forward slightly to increase inertia and forward momentum when going downhill. The rider should stop providing force for speeding up to recover some energy.

In the end, the bicycle's contact with the road itself creates two forms of friction called rolling resistance and friction loss. As a bicycle's tires move on the road, they flatten and expand as they roll. A poorly maintained power train can steal 10% of your power. Fortunately, this is the easiest part of the bike to improve. We are supposed to use good quality tires and take a moment to clean and lubricate your bike chain. We will gain a chariot that moves like lightning

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